MATH 1050 - EXAM 1 REVIEW

1. $3(y-1)^2+5(y-1)+2=0$ $\quad u=y-1$

$3u^2+5u+2=0$

$3u^2+2u+3u+2=0$

$(3u^2+2u)+(3u+2)=0$

$u(3u+2)+1(3u+2)=0$

$(u+1)(3u+2)=0$

3(2)=6	5
1, 5	6
2, 3	5

$u+1=0$ or $3u+2=0$

$u=-1$ or $3u=-2$

$u=-2/3$

$-1=y-1$ or $-2/3=y-1$

$\boxed{y=0}$ $\quad$ $\boxed{y=1/3}$

2. $x+4x^{1/2}-12=0$ $\quad u=x^{1/2}$ $\quad$ Remember, $x^{1/2}=\sqrt{x}$.

$u^2+4u-12=0$

$(u+6)(u-2)=0$

$u=-6$ or $u=2$

$x^{1/2}=-6$ or $x^{1/2}=2$

$(x^{1/2})^2=(-6)^2$ or $(x^{1/2})^2=2^2$

$x=36$ or $x=4$

Since you squared both sides, you *must* check your solutions

Check: $36+4(36)^{1/2}-12\overset{?}{=}0$ $\qquad$ $4+4(4)^{1/2}-12\overset{?}{=}0$

$36+4(6)-12\overset{?}{=}0$ $\qquad$ $4+4(2)-12\overset{?}{=}0$

$48\neq 0$ $\qquad$ $0=0$ ✓

$\boxed{x=4}$ is the only real solution

3. Sorry - I meant this one to come out nicely, but I made a mistake with my negatives, so it is nasty.

$x^6-9x^3-8=0$ $\quad u=x^3$

$u^2-9u-8=0$

$$u=\frac{9\pm\sqrt{(-9)^2-4(1)(-8)}}{2(1)}=\frac{9\pm\sqrt{113}}{2}$$

$$x^3=\frac{9+\sqrt{113}}{2} \text{ or } x^3=\frac{9-\sqrt{113}}{2}$$

$$\boxed{x=\sqrt[3]{\frac{9+\sqrt{113}}{2}} \text{ or } x=\sqrt[3]{\frac{9-\sqrt{113}}{2}}}$$

4. $5x+2y=10$

x-int: $5x+2(0)=10$
$(y=0)$
$5x=10$
$\boxed{x=2 \quad (2,0)}$

y-int: $5(0)+2y=10$
$(x=0)$
$2y=10$
$\boxed{y=5 \quad (0,5)}$

5. $4x^2+9y=36$

x-ints: $4x^2+9(0)=36$
$(y=0)$
$4x^2=36$
$x^2=9$
$\boxed{x=\pm 3 \quad (3,0) \,\&\, (-3,0)}$

y-int: $4(0)^2+9y=36$
$(x=0)$
$9y=36$
$\boxed{y=4 \quad (0,4)}$

6. $y=x^3-2x^2$

x-ints: $0=x^3-2x^2$
$(y=0)$
$0=x^2(x-2)$
$\boxed{x=0 \text{ or } x=2 \quad (0,0) \,\&\, (2,0)}$

y-int: $y=0^3-2(0)^2$
$(x=0)$
$\boxed{y=0 \quad (0,0)}$

7. $y^2=x+5$

x-axis: replace y by -y
$(-y)^2=x+5$
$y^2=x+5$ ✓

y-axis: replace x by -x
$y^2=-x+5$ no

origin: replace x by -x & y by -y
$(-y)^2=-x+5$
$y^2=-x+5$ no

$\boxed{\text{symmetric around the x-axis}}$

8. $4x^2 - y^2 = 16$

x-axis: $4x^2 - (-y)^2 = 16$

$4x^2 - y^2 = 16$ ✓

y-axis: $4(-x)^2 - y^2 = 16$

$4x^2 - y^2 = 16$ ✓

origin: $4(-x)^2 - (-y)^2 = 16$

$4x^2 - y^2 = 16$ ✓

$\boxed{\text{symmetric around the x-axis, the y-axis, \& the origin.}}$

9. $y = \dfrac{3x}{x^3+5}$

x-axis: $-y = \dfrac{3x}{x^3+5}$

$y = \dfrac{-3x}{x^3+5}$ no

y-axis: $y = \dfrac{3(-x)}{(-x)^3+5}$

$y = \dfrac{-3x}{-x^3+5}$ no

origin: $-y = \dfrac{3(-x)}{(-x)^3+5}$

$-y = \dfrac{-3x}{-x^3+5}$ $\quad y = \dfrac{3x}{-x^3+5}$ no

$\boxed{\text{no symmetry}}$

10. $f(x) = \dfrac{x-2}{x^2+5}$ $\quad f(-2) = \dfrac{-2-2}{(-2)^2+5} = \dfrac{-4}{4+5} = \boxed{\dfrac{-4}{9}}$

11. $f(x) = -2x^2 + x - 1$ $\quad f(4) = -2(4)^2 + 4 - 1$

$= -2(16) + 3$

$= -32 + 3 = \boxed{-29}$

12. $f(x) = \dfrac{2x}{x^2-7}$ $\quad f(-x) = \dfrac{2(-x)}{(-x)^2-7} = \boxed{\dfrac{-2x}{x^2-7}}$

13. $f(x) = 3x^2+5x-7$

$f(x+1) = 3(x+1)^2+5(x+1)-7$
$= 3(x^2+2x+1)+5x+5-7$
$= 3x^2+6x+3+5x+5-7$
$= \boxed{3x^2+11x+1}$

14. $f(x) = \dfrac{5x}{x^2-49}$ Domain: $x^2-49 \neq 0$
$(x+7)(x-7) \neq 0$
$x \neq -7$ $x \neq 7$
$\boxed{\{x \mid x \neq -7 \text{ or } 7\}}$

15. $f(x) = \sqrt{15-3x}$

D: $15-3x \geq 0$
$15 \geq 3x$
$5 \geq x$

$\boxed{\{x \mid x \leq 5\} \text{ or } (-\infty, 5]}$

16. $f(x) = -2|x|+8$ Domain: $\boxed{\mathbb{R}}$

17. $f(x) = \dfrac{2x-3}{x+5}$ $g(x) = \dfrac{2-x}{x+5}$

$(f+g)(x) = f(x)+g(x) = \dfrac{2x-3}{x+5} + \dfrac{2-x}{x+5}$

$= \dfrac{2x-3+2-x}{x+5} = \boxed{\dfrac{x-1}{x+5}}$

Domain: $x+5 \neq 0$ $\boxed{\{x \mid x \neq -5\}}$

18. $f(x) = \sqrt{2x}$ $g(x) = 2x-7$

$\left(\frac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \boxed{\dfrac{\sqrt{2x}}{2x-7}}$

Domain: $2x \geq 0 \Rightarrow x \geq 0$
$2x-7 \neq 0$
$\Rightarrow 2x \neq 7$ $x \neq 7/2$

$\boxed{\begin{array}{l}\{x \mid x \geq 0 \text{ \& } x \neq 7/2\} \\ [0, 7/2) \cup (7/2, \infty)\end{array}}$

19. $f(x) = x^2 + 2x$

$f(x+h) = (x+h)^2 + 2(x+h)$

$-f(x) = -(x^2 + 2x)$

$$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 + 2(x+h) - (x^2+2x)}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 \cancel{+2x} + 2h \cancel{-x^2} \cancel{-2x}}{h}$$

$$= \frac{2xh + h^2 + 2h}{h} = \frac{\cancel{h}(2x + h + 2)}{\cancel{h}}$$

$$= \boxed{2x + h + 2}$$

20. $f(x) = 7x - 8$

$f(x+h) = 7(x+h) - 8$

$-f(x) = -(7x - 8)$

$$\frac{f(x+h) - f(x)}{h} = \frac{7(x+h) - 8 - (7x-8)}{h}$$

$$= \frac{\cancel{7x} + 7h \cancel{-8} \cancel{-7x} \cancel{+8}}{h} = \frac{7h}{h} = \boxed{7}$$

21. Function
Domain: $\mathbb{R}$
Range: $(0, \infty)$
y-int: $(0, 2)$
no x-ints
no symmetry

22. Function
Domain: $\mathbb{R}$
Range: $[-4, \infty)$
x-ints: $(-2,0)$, $(0,0)$, $(2,0)$
y-int: $(0,0)$
symmetric around y-axis

23. $f(1)$ ← What is y when $x=1$?

$\boxed{f(1)=16}$

24. For what x is $f(x)=0$? ← Where is y zero?

$f(x)=0$ at $\boxed{-3, 0, \& 2}$

25. Symmetric around y-axis: $\boxed{\text{even}}$

26. symmetric around origin: $\boxed{\text{odd}}$

27. $f(x)=\frac{2x}{3x^2-5}$

$f(-x)=\frac{2(-x)}{3(-x)^2-5}=\frac{-2x}{3x^2-5}$ not even

$-f(x)=-\left(\frac{2x}{3x^2-5}\right)=\frac{-2x}{3x^2-5}$ $f(-x)=-f(x)$ $\boxed{\text{odd}}$

28. $f(x)=\frac{|x|}{x^2}$

$f(-x)=\frac{|-x|}{(-x)^2}=\frac{|x|}{x^2}$ $\boxed{\text{even}}$

29. Intervals on which function is increasing, decreasing & constant. (Between what x-values)?

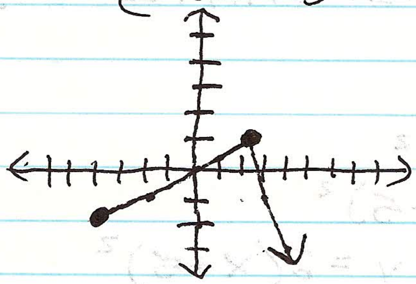
Increasing: $(-4,-3)\cup(2,\infty)$
Decreasing: $(-3,0)$
Constant: $(0,2)$

30. Numbers at which f has local maxima (the x-values). What are local maxima (the y-values?)

$$\boxed{\begin{array}{l}\text{Local maxima @ } x=-1 \text{ \& } x=1 \\ \text{Local maximum: } 4\end{array}}$$

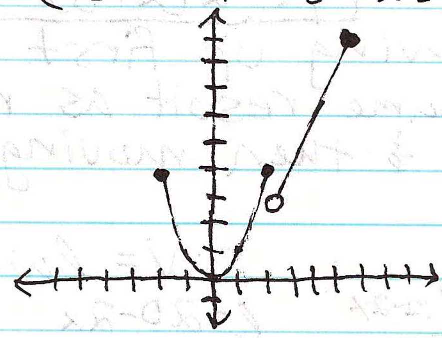
31. $f(x) = \begin{cases} \frac{1}{2}x & \text{if } -4 \le x < 2 \\ -2x+5 & \text{if } x \ge 2 \end{cases}$

$\frac{1}{2}(-4) = -2$
$\frac{1}{2}(2) = 1$
$-2(2)+5 = 1$



32. $f(x) = \begin{cases} x^2 & \text{if } -2 \le x \le 2 \\ 2x-1 & \text{if } 2 < x \le 5 \end{cases}$

$2(2)-1 = 3$
$2(5)-1 = 9$



33. $f(x) = |x-3|$
parent: $y = |x|$
shift right 3

see graph paper

34. $f(x) = 3\sqrt[3]{x}$
parent: $y = \sqrt[3]{x}$
vertical stretch by 3
(multiply y-coord by 3)

see graph paper.

35. $f(x) = \sqrt{-x}$

parent: $y = \sqrt{x}$ see graph paper

reflection over y-axis

36. ~~square~~ square root function shifted ↓ 2

$\boxed{f(x) = \sqrt{x} - 2}$

37. reciprocal function shifted ← 3

$\boxed{f(x) = \frac{1}{x+3}}$

38. Parent: $y = x^2$

← 5 : $y = (x+5)^2$

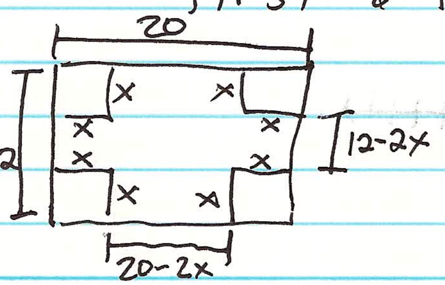
vert stretch by 2: $y = 2(x+5)^2$

↑ 7: $y = 2(x+5)^2 + 7$

reflect across x: $y = -(2(x+5)^2 + 7)$

$\boxed{y = -2(x+5)^2 - 7}$

notice: moving up first then reflecting has same result as reflecting first & then moving down 7.

39. 

$V = lwh$

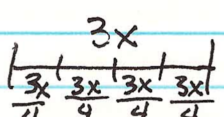
$l = 20-2x \quad w = 12-2x \quad h = x$

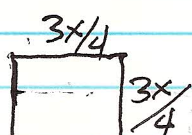
$V(x) = x(20-2x)(12-2x)$

$V(x) = x(240 - 40x - 24x + 4x^2)$

$V(x) = x(4x^2 - 64x + 240)$

$\boxed{V(x) = 4x^3 - 64x^2 + 240x}$

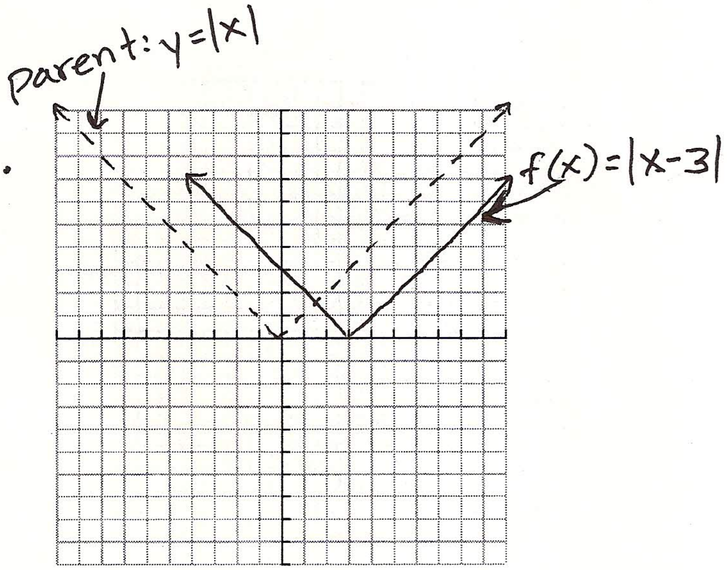
40. 

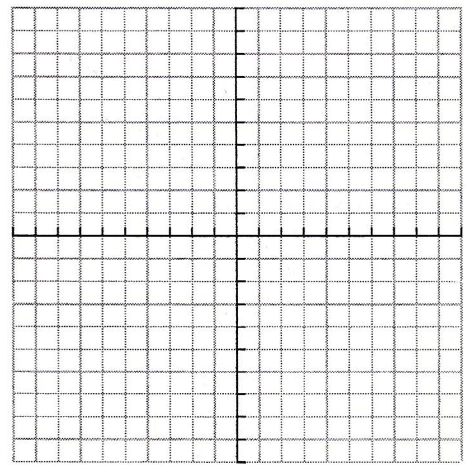


$A(x) = \left(\frac{3x}{4}\right)\left(\frac{3x}{4}\right)$

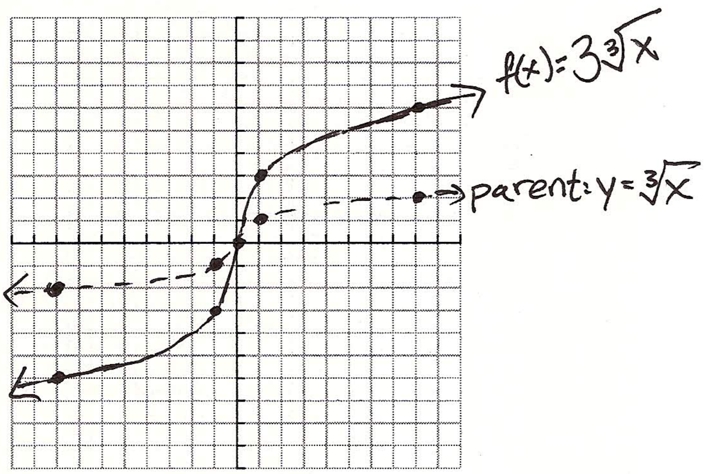
$\boxed{A(x) = \frac{9x^2}{16}}$

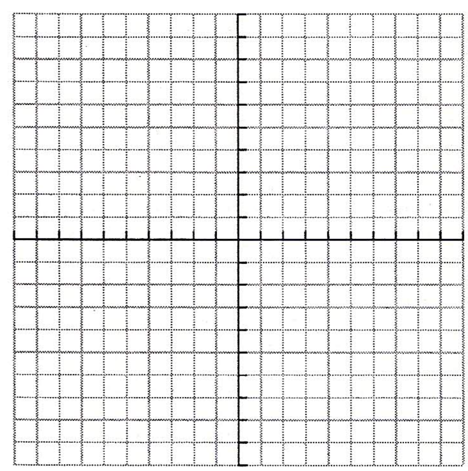
33.





34.





35.